Prove : $\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$

for $n \in \mathbb{Z}^+$.

For proof by induction we need to do the following:

1) Show true for $n = 1$

2) Assume true for $n = k$

3) Show true for $n = k+1$
given true for $n = k$.

Then we can see it is true for $n = 1, 2, 3, \ldots$

$n = 1$: $\sum 1^2 = 1 = \frac{1}{6} 1.2.3 = \frac{6}{6} = 1$.

Assume true @ $n = k$: $\sum_{r=1}^{k} r^2 = \frac{1}{6} k(k+1)(2k+1)$

Show true ∂ $n = k+1$.

$$\sum_{r=1}^{k+1} r^2 = \sum_{r=1}^{k} r^2 + (k+1)^2$$

$$= \tfrac{1}{6} k(k+1)(2k+1) + (k+1)^2$$

(as true ∂ $n=k$)

$$= (k+1)\left\{ \tfrac{1}{6} k(2k+1) + (k+1) \right\}.$$

$$= (k+1)\left(\tfrac{1}{3}k^2 + \tfrac{k}{6} + k + 1 \right).$$

$$= (k+1)\left(\tfrac{1}{3}k^2 + \tfrac{7}{6}k + 1 \right)$$

$$= \tfrac{1}{6}(k+1)(2k^2 + 7k + 6)$$

$$= \tfrac{1}{6}(k+1)(2k+3)(k+2)$$

$$= \tfrac{1}{6}(k+1)(k+2)(2(k+1)+1)$$

ie true for $n = k+1$.

So true for $n=1$, & $n=k \Rightarrow n=k+1$

So true for $n=1,2,\ldots$ by induction.