

$$8^y = 4^{2x+3} \quad (i)$$

$$\log_2 y = \log_2 x + 4. \quad (ii)$$

Solve the simultaneous equations.

$$(i): \log_2 8^y = \log_2 4^{2x+3} \quad (\text{taking logs of both sides})$$
$$y \log_2 8 = (2x+3) \log_2 4. \quad (\text{as } x \log y = \log y^x)$$

$$y = \frac{(2x+3) \log_2 4}{\log_2 8}. \quad (*)$$

In (ii) we notice that the term 4 can be written with $\log_2 (4 = \log_2 16)$. $\therefore$

$$\therefore \log_2 y = \log_2 x + 4$$

$$= \log_2 x + \log_2 16$$

$$\Rightarrow \log_2 y = \log_2 16x. \quad (\text{as } \log a + \log b = \log ab).$$

$$\text{So : } y = 16x. \quad (\text{removing } \log_2)$$

$$\frac{(2x+3)\log_2 4}{\log_2 8} = 16x. \quad (\text{substituting from } (*))$$

$$\therefore (2x+3)\log_2 4 = 16x\log_2 8.$$

$$\therefore 3\log_2 4 = 16x\log_2 8 - 2x\log_2 4.$$

$$\therefore 3\log_2 4 = x(16\log_2 8 - 2\log_2 4).$$

$$\Rightarrow x = \frac{3\log_2^2 4}{16\log_2^3 8 - 2\log_2^4 4}.$$

$$= \frac{3 \times 2}{16 \cdot 3 - 2 \cdot 2} = \frac{6}{48 - 4} = \frac{6}{44} = \frac{3}{22}$$

$$\therefore y = 16x = 16 \times \frac{3}{22} = \frac{8 \times 3}{11} = \frac{24}{11}$$