

$$\int_1^{e^2} x^3 \ln x \, dx$$

$$\int u \, dv = uv - \int v \, du. \quad (\int \text{ by parts})$$

$$\text{Set } u = \ln x \quad dv = x^3 \, dx$$

$$du = \frac{1}{x} \, dx \quad v = \frac{1}{4} x^4$$

$$\text{So } \int = \left[\frac{1}{4} x^4 \ln x \right]_1^{e^2} - \int_1^{e^2} \frac{1}{4} x^4 \times \frac{1}{x} \, dx.$$

$$= \left[\frac{1}{4} e^8 \ln e^2 - 0 \right] - \frac{1}{4} \int_1^{e^2} x^3 \, dx$$

$$= \frac{1}{2} e^8 - \frac{1}{4} \left[\frac{1}{4} x^4 \right]_1^{e^2}$$

$$= \frac{1}{2} e^8 - \frac{1}{16} (e^8 - 1)$$

$$= \frac{1}{2} e^8 - \frac{1}{16} e^8 + \frac{1}{16}.$$

$$= \frac{7}{16} e^8 + \frac{1}{16}$$

$$\text{So } a = \frac{7}{16}, \quad b = \frac{1}{16}.$$