12 The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -3 \\ 4 \end{pmatrix}$. The plane Π has equation $\mathbf{r}.\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 4$.

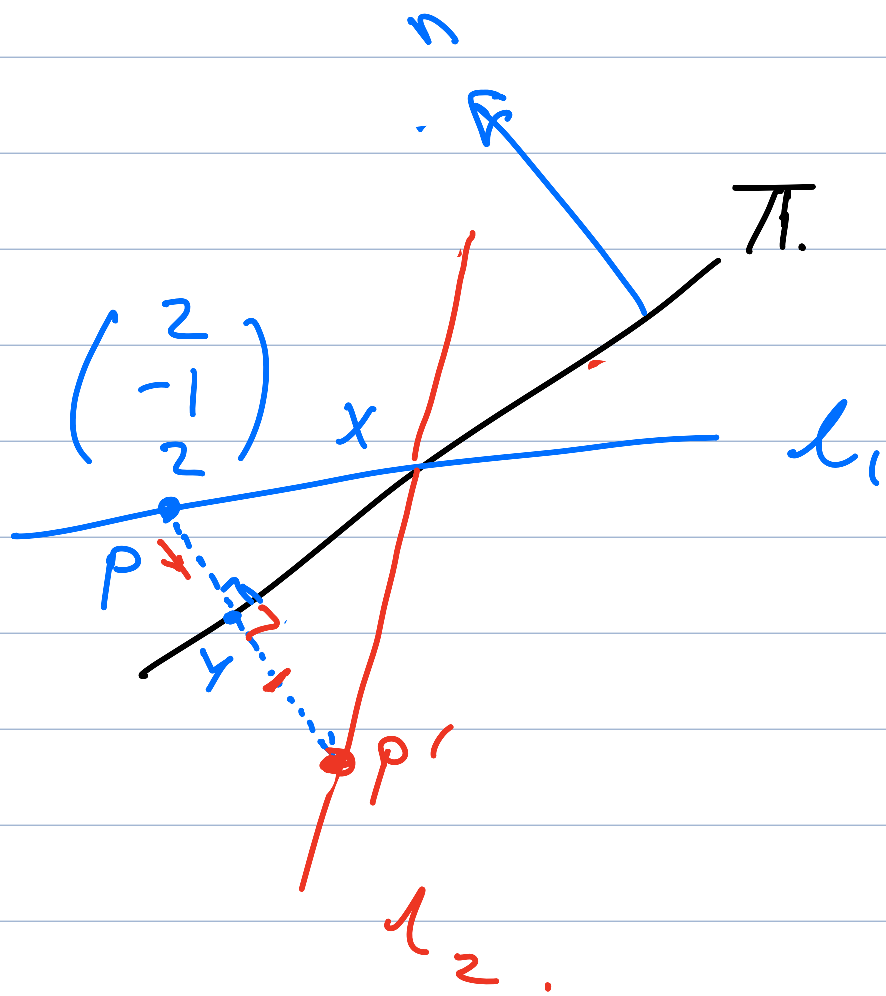
The line l_2 is the reflection of line l_1 in the plane Π.
Find a vector equation of the line l_2. **(7**

12.

$$l_1 : r = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -3 \\ 4 \end{pmatrix}$$

$$\Pi : \quad r . \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 4$$

A brief diagram helps to map out the approach :



We look to find

→ point of intersection between plane Π & l_1 (point x)

→ point of reflection of another point along l_1 in Π (point p')

So l_1 & Π meet when:

i.e $\begin{pmatrix} 2+4\lambda \\ -1-3\lambda \\ 2+4\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 4.$ Some λ

$$(2+4\lambda) - (-1-3\lambda) - (2+4\lambda) = 4.$$

$$3\lambda + 1 = 4.$$

$$\lambda = 1$$

So point X is $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \\ 4 \end{pmatrix}$

$$= \begin{pmatrix} 6 \\ -4 \\ 6 \end{pmatrix}.$$

X must lie on l_2, so l_2

has vector equation:

$$l_2: \quad r = \begin{pmatrix} 6 \\ -4 \\ 6 \end{pmatrix} + \mu z .$$

We need another point on l_2.

in order to find z.

Take the point $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ & find

its reflection in the plane.

line $\overrightarrow{PP'}$ has equation:

normal to plane.

$$r = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \underline{\alpha} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

We can find where P hits the plane in the reflection (point Y) by:

$$\begin{pmatrix} 2+\alpha \\ -1-\alpha \\ 2-\alpha \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 4.$$

$$2+\alpha+1+\alpha-2+\alpha = 4.$$

$$3\alpha = 3, \quad \alpha = 1.$$

So P hits the plane at $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = Y$

So point P' is

$$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 3-2 \\ -2-(-1) \\ 1-2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$$

Now we can find z (direction vector of l_2) as $\overrightarrow{XP'} = P' - X$

$$= \overset{P'}{\overset{\shortparallel}{\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}}} - \overset{X}{\overset{\shortparallel}{\begin{pmatrix} 6 \\ -4 \\ 6 \end{pmatrix}}} = \begin{pmatrix} -2 \\ 1 \\ -6 \end{pmatrix}.$$

So $l_2 : \ r = \overset{X}{\overset{\shortparallel}{\begin{pmatrix} 6 \\ -4 \\ 6 \end{pmatrix}}} + \mu \begin{pmatrix} -2 \\ 1 \\ -6 \end{pmatrix}^{= \overrightarrow{XP'}}.$

OR

alternatively: $l_2 = \overset{P'}{\overset{\shortparallel}{\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}}} + \mu \begin{pmatrix} 6-4 \\ -4+3 \\ 6-0 \end{pmatrix}^{= \overrightarrow{P'X} = X - P'}$

$$= \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$